

## *Singular Terms and Sentential Sign Designs*

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What constraints does the conceptual role played by singular terms put on what sort of sign design can play that role? The answer to be defended here is that singular terms need not be linguistic expressions at all. Sub-sentential syntactic structure does not require the occurrence of distinguishable *components* of sentential sign designs. More generally, representations that are used in such a way that they ought to be understood semantically and syntactically as predications classifying objects according to their properties and relations need not be compounded out of predicate-representations and term-representations. To say that a particular singular term or predicate occurs in a sentence is to say something about the way the use of that sentence relates to the use of other sentences. The occurrence of a singular term in a sentence never consists in the presence of a distinguishable subexpression or component of a representation, though it may be marked by it.

The issue of what it is for a term to occur in a sentence is an important aspect of the study of the conceptual role played by singular terms. It must be addressed by any general theory of representation. For it bears directly on the question of what conditions must be met in order correctly to discern the occurrence of subsententially complex representations in the functioning of a system, for instance, a dolphin brain, an extraterrestrial community, or a computer. The effect of the thesis maintained here is to loosen substantially the requirements for attributing representations that exhibit terms and predicates, compared to the requirements imposed by restricting attention to grammars understood compositionally. Further, the account developed below specifies the conceptual role of subsentential complexity entirely in terms of relations among representings, apart from consideration

of their potential relations to what is represented. In this way it can contribute to the understanding of what it is for a singular term to "purport to refer to exactly one object," as Quine's suggestive indication of this conceptual role has it.<sup>1</sup> The issue of what sort of thing singular term occurrences are also matters in an obvious way for theories of the relation of representing to what is represented. The proper conception of the semantic relations between term occurrences and what those occurrences refer to clearly depends on a proper conception of the relata. Theories of the causal, historical, or informational transmission of semantic links to an object must be somewhat sensitive to the specification of the linguistic items that stand at the end of such chains. In this case the effect of the thesis of the present work is to broaden the conception of these termini, from term tokenings thought of as utterances or inscriptions of distinguishable subsentential expressions, to term tokenings thought of as features of the use of sentence tokenings in relation to other sentence tokenings.

The question of the nature and status of singular terms to which this essay is directed concerns the relation between the physical composition of the sign designs, which are the actual vehicles of linguistic significance, and the exhibition by those sign designs of the syntactic subsentential structure of singular terms and predicates, which must be discerned in order to understand the significance of such sign designs. 'Sign design' is used to indicate the physical, spatio-temporally located aspect of linguistic expressions, the utterances or inscriptions that are actually produced by the performances that are the practices constituting the use of linguistic expressions. The study of these concrete products of utterings and inscribings of abstract linguistic expressions may be called 'orthography,' in a very general sense. Usually that term is restricted to the study of the proper way of writing words by constructing them out of alphabetic characters. The more general usage to be employed here looks beyond words to all linguistic expressions, and beyond alphabetic concatenation to any mode of construction or formation. The writing whose proprieties are at issue according to the etymology of the word is to be understood to be any sort of uttering of linguistic expressions, whether the products be sounds, visible marks, scents, or voltage spikes.

The syntactic concern out of which the question arises is not solely a concern with the best way to generate all and only the well-formed expressions of a language. It is rather a matter of *semantically significant* syntax. An expression or sentence-feature counts as a singular term in the sense relevant here only if it contributes appropriately to the content or meaning

of sentential performances containing or exhibiting it. The 'it' in "It is raining," and the 'sake' of "I did it for her sake," behave syntactically like terms in one sense, but make quite a different sort of semantic contribution than expressions functioning as genuine singular terms. Narrowly syntactic distinctions and assimilations are to be considered only insofar as they help explain differences in semantic function. The result reported here ought accordingly to be of interest to cognitive scientists who must be concerned with the relations between causally significant features of representing sign designs, on the one hand, and the semantically significant syntactic structures they can exhibit in concert with their fellows, on the other.

To say that singular terms and predicates must be identified by the sort of contribution their occurrence makes to the meaning or content of the sentence-tokenings in which they occur is to say that subexpressions or features of sentences count as items of these grammatical categories in virtue of being *used* in a particular way. As features of sign designs are of interest only insofar as they have syntactic and lexical significance, and as syntactic classifications must ultimately show their bearing on attributions of semantic content, so semantic classifications must ultimately show their bearing on pragmatics, on accounts of the appropriate use of the expressions they classify. For many sorts of things might be systematically associated with linguistic expressions: abstract or concrete objects and sets of such objects, model structures and matrices, possible worlds and sets of them, recognitive and justificatory procedures, histories of actual utterance, or what have you. But the claim that displaying such an association amounts to offering a *semantic* interpretation of the expressions must be backed up by showing how the appropriate use of the expressions in question is illuminated or explained by the association. Orthography answers to syntax, syntax answers to semantics, and semantics answers to pragmatics. To adopt this explanatory constraint is not to impose the ultimately instrumentalist 'manifestability' requirement that every property or distinction attributed to expressions, for instance by semantic theory, must correspond to some property or distinction observable in the use of those expressions. The relation of theory to what it explains may be much looser than that. The requirement is only that what is attributed in the vocabulary of syntax help explain what is attributed in the vocabulary of semantics, and that this help explain in turn what is attributed in the vocabulary of pragmatics or use. The present task is to investigate the constraints that are put on the sign designs that instantiate or exhibit singular

terms and predicates, by the use of sentences in virtue of which they are properly understood as exhibiting such semantically significant subsentential syntactic structure.

## I

The best way to approach the issue concerning the relation between sign designs and semantically significant subsentential syntax that is being pursued here passes through three familiar ways of conceiving such syntax. They are: general categorial grammars with associated semantics, as developed by Montague and Lewis; functional syntax as conceived by Frege; and the 'predicateless' syntax elaborated by Sellars under the heading of 'Jumblese'. Examining the conceptions and motivations that distinguish these lines of thought will permit the assembly of the raw materials for an account that is more liberal than any of them in its demands on the physical expression of subsentential syntax.

A categorial grammar can be conceived as beginning with a number (perhaps only one) of basic syntactic categories.<sup>2</sup> For present purposes, as for Lewis', it suffices to consider only two, {S,T}, sentences and singular terms. This set of basic categories then generates a potentially infinite set of derived categories, according to a simple functional architecture. If X,Y are categories (basic or derived), then  $(X>Y)$  is also a category. It corresponds to a syntactic function that, when given an argument of category X, returns a value of category Y. (The general definition would allow a set  $X_1...X_n$  of arguments.) Thus one-place predicates, in the simplest case, can be thought of as belonging to the derived category of  $(T>S)$ , that is, as items that apply to terms, items from one basic category, and that yield then sentences, items from the other basic category. In general, a derived category is specified by specifying the categories (basic or derived) of the items it applies to, and the category of the items that result from such application. Term-forming functors such as 'the father of ...' are accordingly understood to be of derived category  $(T>T)$ . Adverbs, which turn predicates like 'red' into predicates like 'bright red' are represented by the derived category  $((T>S)>(T>S))$ . Similarly, term-forming operators like 'the father of ...' are represented as  $(T>T)s$ , modal operators that turn "3 is twice 6" into "It is necessary that 6 is twice 3" are  $(S>S)s$ , quantifiers are  $((T>S)>S)s$ , and so on. The simple structure of basic and derived categories supplies a rich variety of 'parts of speech' or kinds of syntactic combinatory role for a grammar to work with.<sup>3</sup>

Conceiving syntactic and semantic structure as explicitly parallel according to this functional model has a number of advantages. It includes a uniform procedure for generating a wealth of parts of speech. The basic syntax need only provide a single mode of appropriate combination of expressions of all categories — anything adequate to express functional application will do, for instance concatenation. The sort of semantic significance that ought to be associated with each derived category can be determined easily and in a uniform fashion. Once one has settled the sort of semantic interpretant to be associated with items falling into the basic categories, the semantic interpretants associated with items from the derived categories come for free. For instance, once one has decided how to interpret sentences and singular terms, one has thereby settled (committed oneself to) an interpretation of one place predicates. They must be, or at least determine, functions from whatever interpretant one assigned to terms to whatever interpretant one assigns to sentences. If sentences are interpreted as sets of possible worlds and singular terms are interpreted as individuals repeatable across those worlds, then predicates must be functions from individuals to sets of possible worlds. Adverbs must be semantically interpreted by (something that determines) functions from such functions from individuals to sets of worlds to such functions. And so on, for all of the derived categories. The theorist must choose semantic interpretants, e.g. abstract objects, for the basic categories only. The categorial machinery then grinds out corresponding kinds of interpretants for the derived categories, such as adverbs, quantifiers, and modal operators, automatically. Given the functional architecture, the theorist can concentrate on interpreting the basic categories, and let the structure determine what sorts of interpretants are appropriate for other categories. So this abstract framework is equally applicable for semantic approaches that assigns quite different sorts of interpretants to items of the basic categories, for instance one that seeks to associate each singular term with a practical means of recognizing an object, and each sentence with a set of conditions under which its truth would be ascertainable.

The elegant categorial grammars, with their associated automatic semantics, are the heirs of Frege's original conception of semantically significant subsentential syntactic structure. For Frege the choice of term and sentence as syntactically and semantically basic categories and the association of syntactic and semantic functions with derived categories are not two independent commitments, optionally or accidentally conjoined. They are commitments that presuppose and involve each other. As far as the

formal machinery of categorial grammars is concerned, there is no reason why one should not start with predicates and sentences, rather than terms and sentences, as the basic categories. But Frege argues that it would be a mistake to do so. His reasons and the conception that motivates them furnish another piece to the puzzle being assembled here. He provides, as categorial grammar does not, constraints on associating particular semantically significant syntactic categories with pieces or structures of the expressions or sign designs that incarnate those syntactic categories. For Frege's understanding of the significance of quantification leads him to a principle that associates basic categories with a certain kind of expression, what he called 'complete' ones, and that associates derived categories, corresponding to functions, with structures that are not properly thought of as expressions at all, what he somewhat unhappily called 'incomplete' expressions. He gives a determinate sense to the claim that predicates (as their occurrence is significant for the inferences codified by the use of quantifiers) are not a kind of expression. It is this train of thought that we will follow out to the eventual conclusion that *no* subsentential syntactic and semantic categories need be associated with subsentential *expressions* at all.

The functional decomposition of sentences is one of Frege's discoveries and legacies. His route to it goes through the notion of substitution. We are so accustomed to thinking in terms of very general concepts of functions that it requires something of an effort to recover the basis of these concepts in substitutional relations among sentences. Nevertheless, it is only in this way that one can come to understand the very special notion of subsentential component or structure that such functional analysis gives rise to. The Fregean notion of logical form, of which categorial grammars are an elegant elaboration, depends essentially on distinguishing the *orthographic composition* of a sentence from its *syntactic analysis*. The canonical formal languages addressed by traditional model theory are specified in such a way as systematically to obscure this important distinction, so consideration of the Fregean two stage approach forms a better starting point.

This much is common to Frege and his model-theoretic heirs: A pre-quantificational language is conceived compositionally as consisting of sentences that are built out of simple components that can be specified independently of and antecedently to specification of the sentences that are built out of them. Some expressions, perhaps an infinite number, are stipulated to be singular terms. Other expressions are stipulated to be predicates and functors. These may be called 'simple' predicates (and

functors). Each predicate (and functor) has a particular number of distinct term positions associated with it, its adicity. When a predicate expression is supplemented by an appropriate number of term expressions (perhaps by concatenation, perhaps according to some more complicated structure), the expression that results is a sentence. It is these expressions that are involved in inferences, as premises and as conclusions, and so that are used to express the undertaking of assertional commitment. The inferences associated with the sentence must depend, in part, upon what terms appear in it, and what simple predicate is used in its construction, if the syntactic compositional structure of the sentence is to be semantically significant. The structure that is available to bear such significance is exhausted by the simple components out of which sentences are constructed.

According to Frege, languages for which the sort of compositional subsentential structure just considered is all that is inferentially significant are expressively impoverished. In order to have the expressive power achieved by locutions permitting arbitrary iterations of quantifiers, required for even the simplest mathematics, further structure must be discerned. In particular, predicates must be more finely individuated, with each simple predicate corresponding to many complex predicates. The inferential content of quantified expressions is determined in a uniform way by the contents of their quantificational instances. These are sentences of a form determined by the quantificational claim. The current concern is not the (disjunctive or conjunctive) form of this semantic inheritance from sets of instances, but merely the expressive power to specify and assimilate those instances in virtue of their form. One cannot group sentences correctly into equivalence classes of instances corresponding to various quantificational constructions unless one discerns subsentential structure corresponding to complex predicates, not merely to simple ones.

An example will make clear the difference between simple and complex predicates.<sup>4</sup> In order to distinguish correctly the appropriate inferential antecedents and consequents of a sentence such as "Everyone who influenced another also influenced himself," one must look conjunctively at a set of instances of the form "Goethe influenced Carlyle and Goethe influenced Goethe." Mastering the inferential content of these instances requires being able to distinguish instances of the form of the first conjunct from instances of the form of the second, depending upon whether the influential one is the same one influenced or not, and being able to correlate these accordingly as the influential one is the same in each case. But there is no simple predicate or other component in common to "Goethe influ-

enced Carlyle," and "Tacitus influenced Gibbon," that is not also shared with "Goethe influenced Goethe." And the only simple component the latter shares with "Tacitus influenced Tacitus," it also shares with "Goethe influenced Carlyle." The complex predicate can include, as the simple predicate does not, information concerning the cross-identification of the terms occupying some of its term positions. The significance of quantificational expressions depends upon such information. The syntax of a prequantificational language into which quantificational locutions could be introduced must accordingly individuate expressions finely enough to enable the expression of such information. That is, the syntax must support the discerning of complex subsentential structure, which is not simply a matter of the composition of expressions out of simple separable components.

Frege's account of the syntactic origins of complex subsentential components requires two distinct stages. After the synthetic constructive compositional stage in conceiving simple predicates and singular terms, there is an analytic substitutional one. During this stage complex components are discerned within sentences by considering their substitutional relations to other sentences. Frege's metaphor is initially helpful, though I shall argue that in the end it is misleading. One omits a term from a sentence, to produce a sentence frame or predicate, which when replaced by various terms yields various sentences or instances of the predicate formed by such omission. The mechanism of complex predicate formation by term omission is surprisingly powerful. For instance, it can account for the relation between the complex predicates considered above in "Goethe influenced Goethe," and "Tacitus influenced Gibbon." One complex predicate is formed from "Goethe influenced Goethe," by omitting all occurrences of the term 'Goethe'. This forms a one place predicate, 'influencing oneself'. Another complex predicate can be formed from "Tacitus influenced Gibbon," by omitting first 'Tacitus', and then 'Gibbon', to yield a two place predicate, which will include as instances all sentences that are instances of the predicate 'influenced oneself', among others. The one place predicate may accordingly be understood as a sub-predicate of the two place one. The machinery of predicate formation by term omission also yields novel predicates when applied to sentential compounds. In "Goethe influenced Carlyle and Goethe influenced Goethe," one of the instances of the quantificational claim mentioned above, by omitting all occurrences of 'Goethe' one discerns the complex predicate "influenced Carlyle and influenced himself."

Frege was metaphysically puzzled by predicates, and by functions generally.<sup>5</sup> One reason some of his ontological pronouncements are hard for us to process is that it is hard not to read them as treating complex subsentential structures as though they were simple components, but components of a peculiar sort. Whereas the point is that these complex predicates are not separable parts, but features of or patterns in sentences. Sentences can be understood as "built out of" components of this sort only in a special way. Complex components, unlike simple ones, cannot be specified apart from a specification of sentences exhibiting them. Complex "constituents" are distinguished only by what the Scholastics called a *distinctio rationis*, not by a *distinctio realis*. The latter requires separability. The former requires only comparison. Thus form and matter are only distinct in thought, as we can compare a cube of wood, a cube of marble, and a sphere of marble, calling 'shape' what the first two but not the third share, and 'matter' what the latter two but not the first share. Frege's model of substitution in terms of omission and replacement led him to make needlessly puzzling claims about the "ungesattigkeit" or unsaturatedness of predicates, "objects with holes in them." Much of the conceptual difficulty he wrestled with can be obviated by thinking of predicates as *relational* features of sentences. What difficulty remains is that associated with discerning a structure in order to account for a use or behavior of an expression, that is, with the notion of logical, or better, inferential, form generally, the topic discussed below.

Frege showed how to turn a recipe for understanding single occurrences of quantifiers (which the Medievals already had down pat) into a uniform recipe for understanding arbitrarily nested occurrences of quantifiers. That recipe depends upon conceiving the resulting quantificational sentential expressions as constructed step by step by omission and replacement of singular terms. It essentially depends upon the two stages of syntactic construction already distinguished. First one considers sentences compositionally, in terms of their simple, separable components. These sentences may then have sentential operators applied to them to form sententially compound sentences. Then predicates are formed by omitting terms, some of which may be repeated, from the various sentences that result from the first stage. It is sentences grouped accordingly as they could be taken to exhibit various such complex predicates that are required for the introduction of quantifiers. The question to be asked now is this: For many reasons it seems that the second stage of this process is essential to discerning predicate structure in the sense that matters for quantification. Is

this true of the first stage as well? That is, *must composition from simple components precede analysis into complex patterned predicates?* The complex components discerned within sentences may not, it has been claimed, be conceived of as parts. Does the discrimination of substitutionally complex subsentential structural features presuppose the prior capacity to discriminate simple components out of which the sentence can be taken to be composed?

The question of concern here is whether possession by sentences of the inferential contents in virtue of which they can 'purport' to involve reference to individual objects presupposes that sentential expressions are composed of term expressions that one could sensibly talk of 'omitting'. This question arises concretely for theoretical attributors of substantially alien linguistic practices, say to dolphins or extraterrestrials. Dolphins apparently communicate passively, by altering the configuration of their internal organs. The active 'listeners' bounce sonar waves (to which what covers those organs is transparent) off of the passive 'speakers', and respond differentially depending on features of the echoes. If they produce sentential utterances, it is not likely that these can be parsed as strings of concatenated 'subexpressions'. Do we understand what it would mean to 'omit' from a sentential configuration an occurrence of a certain geometrical attitude of the spleen? Need we? Or consider how one would proceed with developing a theory according to which the extraterrestrials under study communicate by fragrances. Sentences are uttered by producing a particular odor, thereby undertaking an assertional commitment. Must those sentences contain parts that it makes sense to talk about 'omitting', on pain of in-principle relegation to second-class status with respect to expressive power?

The Fregean approach to substitution suggests that there is such a pre-supposition. For that approach depends upon 'omitting' some component from an antecedently specifiable sentence. What is omitted must, it seems, be separable, be a part rather than just a feature. It need not, of course, be a term. Complex sentential operators can be discerned by omission of sentences from compound sentences such as  $\sim(p \& \sim q) \rightarrow (q \rightarrow \sim(\sim p \& s))$ . Nor need the expression from which something is omitted be a sentence. Omitting 'honey' from the definite description 'the land of milk and honey', yields a term-forming operator which when applied to 'cheese' yields a description of Wisconsin. But whether terms or sentences, it is simple components, constituents in fact, that are being omitted. Indeed, this is just what one would expect from the categorial picture, if (T>S)s are to

be formed from Ss by omission of Ts. Complex predicates cannot be conceived as 'omitted', just because they are not parts that can be separated from the sentences exhibiting them. They exist only as patterns of variation. Frege's way of conceiving things requires the contrast between simple constituents, which can be 'omitted', and complex components, which are patterns exhibited by such omission and replacement. Is this a parochial feature of Frege's conception, or an essential feature of any scheme that discerns subsentential structure of the sort of complexity required for assertional explication of inference by introducing quantificational locutions?

The requirement that sentences be discerned is non-negotiable. It is enforced by the dual requirements that syntax be semantically significant and that semantics be pragmatically significant. For the central use of linguistic expressions is *assertion*. And sentences are, by definition, expressions such that in the absence of special circumstances, their free-standing utterance has the pragmatic significance of making an assertion, that is, of undertaking an assertional commitment on the part of the utterer. The pragmatic significance characteristic of assertional commitments is at least a matter of the appropriate grounds and consequences of such commitments, that is, a matter of what else one becomes committed or entitled to by undertaking such a commitment, and of what would commit or entitle one to that commitment.<sup>6</sup> This is to say that the semantic contents associated with assertional commitments must be *inferentially* articulated. The content of a claim must determine what other contents follow from it, and what contents it follows from, in both committive and permissive senses of 'follows from'.

It was indicated above that quantificational inferential articulation of sentences depends on the possibility of treating sentences as exhibiting the subsentential syntactic structure of singular terms occurring in different positions in complex predicates. Expressions could not be taken to be singular terms, that is to have as their job representing or referring to particular objects, unless those expressions could also appear as subjects of predications. For only so can anything be said (that is, claimed, a move made in the language game) by their use. Quine notoriously discovers the ground of the referential purport of singular terms in their involvement in predications supporting quantificational inferences. One need not accept such involvement as a sufficient condition of referential purportment in order to accept it as a necessary condition. To do so is not to insist that any language with singular terms must have quantificational expressions

in it, but only that any such language must have the term-predicate structures that makes it possible to introduce such expressions. No deictic or anaphoric credentials can make something a singular term if it does not play the right inferential role. For then it cannot combine appropriately with predicates to form sentences, and so cannot be used to say anything. It is for this reason that the focus here is on the inferential presuppositions of the representational role of terms.

In seeing how one might go about preserving the expressive power corresponding to discerning the syntactic occurrence of semantically significant singular terms and predicates while not requiring distinct sub-expressions, a first step is to consider Sellars' imaginary language Jumblese, which elaborates a Tractarian point about predicates.<sup>7</sup> It helps show that talk of 'omission' and 'replacement' is a metaphor too closely tied to one among many possible syntactic instantiations (incarnations) of linguistic structure, and it illustrates how that metaphor of 'omission' and 'replacement' can fail to apply, though the substitutional phenomenon it addresses is still evident. Prequantificational sentence inscriptions consist of singular term inscriptions with various properties. For instance, the claim that the thing denoted by 'a' is red might be expressed by an inscription of an 'a' turned on its side, or twice its normal size. The claim that the thing denoted by 'a' is influenced by the thing denoted by 'b' might be written by putting an 'a' two spaces to the left of a 'b'. Other properties of and relations among objects are expressed by colors, type faces, and other orthographic properties and relations. Such a language highlights the point that the only function of simple predicate expressions is to provide a canonical and easily discriminable set of such orthographic properties and relations, by introducing dispensable auxiliary expressions. Thus, instead of having to print the 'a' in red, we print it to the right of a 'P'. The very existence of simple predicate expressions is a notational convenience, of no ultimate significance for the nature of linguistic expressions in general. What is important is only that sentences be permitted to vary according to certain patterns, e.g., different letters all being able to appear printed in red ink, and the same letter being able to appear printed in various inks.

The task of the second part of this paper is to make sense of the notion of substitution required to define complex predicates (and ultimately expressions of all derived categories), without relying upon any picture of 'term omission' that presupposes separable singular terms. Jumblese is not a fully general case, because it depends upon singular terms being simple

expressions out of which sentences can be compounded. But in fact the formation of complex predicates, which bound variables express and quantifiers (among other equally fundamental locutions such as definite descriptions) depend upon, does *not* depend upon the existence of subsentential constituents of this sort. *Languages can be described in which there are no separable subsentential constituents, but in which sentences nonetheless exhibit features expressing the full subsentential structure of singular terms and complex predicates.* By considering a procedure general enough to explain the nature and significance of such complex subsentential structure in these noncompositional cases, one can hope to understand what such structure is, insulated from distractions provided by the convenience of compositionally incarnated syntax. To do so requires showing the structure that is metaphorically invoked by talk of term omission and replacement. Substitution relations among sentences must be considered in the abstract, in terms of their inferential significance, without the term omission picture of these relations that enforces the compositional picture of the relation between syntactic structure and the sign designs that are its vehicles.

## II

The claim is that one need not be able to find simple, separable constituents or parts of sentential sign designs, out of which the latter have been compounded, in order to discern in them complex subsentential syntactic structure, including the occurrence of both singular terms and complex predicates. The discussion of Frege's discovery of the necessity to discern the occurrence of complex predicates within sentences, over and above the simple predicates by whose means those sentences were generated, introduced a noncompositional sort of subsentential substructure. Complex predicates are semantically significant structures that must be discerned within or attributed to sentences, or into which they must be analyzable, in order to explain the behavior of those sentences in the context of the inferential commitments that will be made assertionally explicit by the use of quantificational locutions. So they are features or respects of similarity exhibited by sentences in virtue of their inferential relations to other sentences. The example of Jumblese showed that a semantically significant syntax could confer the same status even on *simple* predicates, which in that case also need not be separable parts or components of sentential sign designs. What is now required is to extend the example of

Jumblese, to show how even the occurrence of singular terms, the paradigm of 'complete expressions' for Frege, can, like complex predicates, and like the simple predicates of Jumblese, consist of features of or relations between sentences, independently of the existence of distinguishable parts.<sup>8</sup> It is a criterion of adequacy of the present account that no component subsentential expressions at all be required. The sentences involved should be thought of as distinct, but without a significant compositional structure. They could be indexed proposition letters, numerals, fragrances, or dolphin gut-wrenchings.

Singular term usage is a pattern in or feature of sentence usage. The use of expressions in virtue of which they count as sentences is assertion, and asserting is undertaking a commitment whose content is articulated by its inferential relations to other contents of possible assertings. The present task is to say what form of relation among sentences must be derived from patterns of sentence *use* to make significant discerning complex subsentential structure, including singular terms and complex predicates, regardless of what the sign designs are like or how they are constructed.<sup>9</sup> It is to specify the inferential relations atomic sentences must stand in to warrant attributing to them the structure they must have if it is to be possible to introduce quantificational and identify locutions that will permit the assertionally explicit codification of those inferential patterns.<sup>10</sup> Usually, the compositional construction of sentential sign designs out of subsentential sign designs is employed to define substitutional relations of variation and instantiation among sentences, and then those relations are appealed to in specifying inferential relations. The present strategy is to reverse this order of explanation: start with inferential relations, use them to define substitution relations among sentences, and then use these to impute subsentential syntactic structure to expressions. The procedure for identifying the mappings that make this work is to pick a set of necessary conditions on substitution transformations in a syntax instantiated compositionally, and enrich it until those conditions can be treated as sufficient for substitutional behavior. Thus each condition that is imposed on transformations below should first be checked to ensure that it holds in the paradigmatic compositionally based syntax. Then the whole set of such conditions should be checked to see that it is sufficient for the specification of term and complex predicate occurrences in sentential sign designs.<sup>11</sup>

In the basic case of a language without explicit quantification or identity, the term-and-predicate structure of an atomic sentence is inferentially significant according to two different patterns of substitutional inference. The

first form of inference is: Pa, therefore Pb. The other is: Pa, therefore Qa. The first is licensed by an inferential commitment regarding singular terms which, when it is made explicit, will take the form of an identity claim  $a=b$ . The second is licensed by an inferential commitment regarding predicates which, when it is made explicit, will take the form of a quantificational claim  $(x)[Px \rightarrow Qx]$ . Where inferences can be collected according to these patterns, sentences can be identified as containing occurrences of singular terms and predicates, and the identity and quantificational locutions can be introduced. Intuitively, what is needed to be able to identify instances of these term- and predicate-inducing inferential patterns is to be able to tell when one sentence can be considered as resulting from another by substituting one subexpression of a category of simple components (terms or predicates) for another subexpression of the same category — in the case above, 'b' for 'a', or 'Q' for 'P'.<sup>12</sup> Given the goal of eschewing appeal to compositional subsentential structure, this description of substitution is not available. Instead, the strategy must be to begin with a set of mappings that take sentences into sentences, and enforce conditions on them that will suffice to make them behave as though they were proper substitutional transformations.

The basic insight to be elaborated is that whatever one wants to say about the syntactic subsentential structure of singular terms and predicates may be expressed equivalently by means of relations among sentences, relations that can be understood as substitutional relations. Further, syntactic structure expressed in the form of those substitutional relations can be understood in terms of its semantic significance, quite apart from any consideration of the nature or composition of the sign designs to which the syntactic structure is attributed. The fundamental semantic features of sentences in virtue of which such structures can be discriminated are their inferential relations to other sentences. According to the pragmatics I endorse,<sup>13</sup> the proper description of the use of sentences in virtue of which their inferential relations ought to be understood as part of the semantic content of sentences is a specification of the normative proprieties of attributing discursive commitments of various sorts, paradigmatically assertional ones. The semantic content of the claim that p, is articulated in part in terms of the inferential commitments that determine that anyone to whom assertional commitment to p is attributed ought also to be taken to be committed thereby to q, and that anyone who is taken to be committed to r ought also to be taken to be committed thereby to p. For present purposes access may be supposed to all such proprieties of conditional

attribution of assertional commitment that govern the practice of some community with respect to a set of sentences. On this basis substitutional relations among sentences are to be picked out that would correspond in the usual compositional instantiation of syntax to substitution of terms for terms. The basic sort of inferential commitment that exploits such term substitutions is identity commitments, commitments of the sort that when made explicit as the content of assertional commitments take the form of identity claims. For the pragmatic significance of such commitments is precisely as substitution licenses — in the compositional model, anyone who attributes to someone an identity commitment regarding terms  $t$  and  $t'$  is thereby committed to attributing to that individual assertional commitment to  $P(t')$  just in case he is committed to attributing commitment to  $P(t)$ , where these sentences are related by just the sort of substitutional relation on which the discrimination of syntactic structure is to be based.<sup>14</sup>

The idea is then that if one can pick out attributions of inferential commitments corresponding to term identities, then one can specify term-substitutional relations among sentences by looking at the significance of those identity commitments for preserving, that is extending, attributed assertional commitments. To pick out the identificatory inferential commitments, it is possible to exploit the fact that they are *symmetrical*, commitments such that anyone to whom they are attributed is thereby taken to be committed to  $p$  if to  $q$ , and to  $q$  if to  $p$ . For in the standard compositional case, term identities license symmetrical term substitution. This necessary condition on identity commitments can be used as the basis for a sufficient one. The requirement that the significance of attributing such a commitment be symmetrical in all contexts of other attributed commitments rules out the cases where I think that if Bill likes hunting, then John will, not because I think Bill is John, but because I think that they like the same sorts of outdoor sports, as well as my being willing to say that anyone committed to Fido's being a dog is committed thereby to Fido's being a mammal. It will not rule out the cases where I think two predicates apply to just the same things, say 'being genetically equipped for a kidney' and 'being genetically equipped for a heart', but those will be filtered out further along in the process [see Appendix, A]. These proprieties of consequentially attributed commitments define relations among sentences that can be represented as mappings. Attention has been restricted to those relations  $f$  for which there is, according to the practices of the community, a corresponding inferential commitment interlocutors can attribute (though the language need not have the expressive resources to permit such com-

mitments to be made explicit, i.e., to be undertaken in the form of *assertional* commitments), such that anyone to whom that commitment is attributed is thereby taken to be conditionally committed to attribute assertional commitment to  $p$  if to  $q$ , and to  $q$  if to  $p$ , whenever  $q$  is related by  $f$  to  $p$ . In the compositionally instantiated syntax, these mappings correspond to simultaneous substitution relations, for instance the one that relates each sentence to the sentences that can result from it by substituting occurrences of 'Goethe' for some occurrences of 'Carlyle', and of 'Gibbon' for some of 'Tacitus' [see Appendix, B].

From now on only the set of relations or transformations extracted in this way from the inferential practices that are to be codified and explained by attributing syntactic structure to the sentences involved is to be considered. The procedure assumes in effect that these transformations are defined on the sentences of an extensional, prequantificational language, and that the relations correspond to substitutions of singular terms for singular terms. Conditions are imposed along the way to ensure that the relations being considered behave enough like the results of genuine term-substitutions instantiated compositionally to guarantee that the construction produces the desired result. From this point on the question is, given only such sentences and a set of relations on them that in fact consisted of all of the relations induced by substitution of terms for terms, is it possible to reconstruct the full subject-predicate structure of the sentences from those transforms alone?

The first bit of syntax that such a set of relations makes available is the ability to group sentences according to the simple predicates they share. The fact that 'Pab' and 'Pcd' exhibit the same simple predicate is expressed in the fact that there is a sequence of term substitutions that turns the one into the other. Say that two sentences are substitutionally *congruent* relative to a set of transforms just in case there is a finite sequence of substitutions that will turn the one into the other [see Appendix, C]. Assuming that there are 'enough' transforms (about which more later), this feature of term-substitution can be exploited to *define* two sentences as sharing a simple predicate just in case they are congruent. Simple predicates can then be identified with congruence classes of sentences.

It is only slightly more complicated to use the substitution transformations to sort the sentences according to the terms that occur in them. The strategy is to represent each singular term by the set of sentences that exhibit it, or in which it occurs. Thus in a compositional syntax, the singular term 'b' is represented by a set of sentences such as {'Pb', 'Qb', 'Rab',

...}. The sentences containing a particular singular term can in turn be picked out as those to which certain transformations apply nonvacuously, as those cases in which substitutions for that term yield new sentences.

To begin with, say that a transform *applies non-vacuously* to a sentence iff the result of applying the transform to the sentence includes some sentence distinct from that to which the transform was applied. Each transform can then be associated with the set of sentences to which it applies non-vacuously. These are the sentences that contain the terms the transform substitutes for. Two transforms that apply nonvacuously to the same set of sentences substitute for the same terms, and two sentences to which exactly the same transforms apply non-vacuously contain the same terms. Given these assimilations, and making some natural assumptions about the behavior of the transforms, it is possible to determine which of the transforms involve substitutions for only a single term, by looking at transforms that apply non-vacuously to sets of sentences that are minimal in having no proper subsets that are all and only the sentences to which some other transform applies non-vacuously [see Appendix, D]. Call these transforms 'single' substitutions, and the rest 'simultaneous' substitutions.

The single substitutions can then be partitioned into equivalence classes accordingly as they non-vacuously apply to just the same sentences. These all substitute for the same term. The term can then be identified with this set of sentences (it would work just as well to identify the term with the class of substitutions that intuitively replace it), and these sentences in turn can be treated as those in which the term occurs. So each sentence can be associated with the set of terms that occur in it. Each single substitution transform can then be indexed by the set of sentences to which it non-vacuously applies, that is, by the term that it substitutes for. By exactly symmetrical considerations assimilating transforms according to the sentences that are the *results* of their non-vacuous applications, the single substitution transforms may then be indexed according to the substituting terms, those they introduce rather than those they eliminate [see Appendix, E].

The simultaneous substitutions can then be indexed according to what terms they substitute for what other terms. What has been said so far suffices to associate with each simultaneous transform a set of sentences it non-vacuously applies to, and a set of sentences it non-vacuously results in. Further, because of the way single substitutions were discriminated from simultaneous ones, these sets can be decomposed and represented as unions of sets of sentences that are terms (or what is the same thing here,

that are all those sentences in which a particular term occurs). This suffices to define the set of terms that the transform substitutes for, and the set of substituting terms. But it does not yet explain how to match them up. The index needed is a set of ordered pairs, with the first element drawn from the set of terms substituted for by the transform, and the second drawn from the set of terms the transform substitutes for them. The index of the transform taking "Goethe influenced Carlyle" into "Tacitus influenced Gibbon" is to be  $\{\langle\text{Goethe}, \text{Tacitus}\rangle, \langle\text{Carlyle}, \text{Gibbon}\rangle\}$ . The question is how to tell that it should not be  $\{\langle\text{Goethe}, \text{Gibbon}\rangle, \langle\text{Carlyle}, \text{Tacitus}\rangle\}$  instead, given that nothing can as yet be said of what *position* a term occurs in. Indeed, it is because of the need to define such positions that it is important to be able to index the simultaneous substitutions by specifying what terms are substituted for what others by each mapping.

The key is to separate the effects of the different individual substitutions comprised by the simultaneous substitution. To do so, look at non-vacuous applications of the transform to sentences in which only a single one of the terms substituted for by the simultaneous transform occurs. In the example above, the proper indexing of the transform in question can be ascertained by applying it to "Goethe influenced Schiller". Since the only non-vacuous result will contain the term 'Tacitus' and not 'Gibbon', the transform must substitute the former for 'Goethe', and not the latter. If there are enough sentences, and the results of this isolation procedure do not depend on what sentence containing only one of the substituted for terms is chosen, then by this procedure each simultaneous transform can be unambiguously indexed with a set of pairs of substituted for and substituting terms. Formulating the indexing scheme for simultaneous substitutions motivates the statement of natural requirements on their behavior, and on the relations between single and simultaneous substitutions [see Appendix, F].

Although the set of sentences and the set of terms may be infinite, in the standard case only a finite number of term occurrences are permitted in each sentence. The apparatus introduced so far can deal with terms and predicates in the more general sense, but it is straightforward to introduce the restriction to more familiar notions. Two things are required: that each sentence have only a finite number of terms occurring in it, and that each of these terms occur only a finite number of times in that sentence. The first is straightforward, for it has been shown how to say what terms occur in a sentence. The required stipulation is that when each sentence is associated with a set of sets of sentences (standing for terms) by the following

procedure, the result must be a finite set. First form equivalence classes of single substitution transforms accordingly as they apply non-vacuously to just the same sentences. These all substitute for the same term, but the substituting term is different in each case. These equivalence classes of transforms define overlapping classes of sentences, with two sentences assimilated in case all the members of some equivalence class of transforms apply non-vacuously to both. The sets of sentences that result are the singular terms, and their members are the sentences in which those terms occur. Associate each sentence with all the terms occurring in it, that is, with all the sets of sentences that are terms that it is a member of. There must only be a finite number of these.

Specifying that each of the finite number of distinct terms occurring in a sentence occurs only a finite number of times might seem to be more difficult. How, on the present approach, can one make sense of the notion of a term's occurring more than once in a sentence? For there is nothing corresponding to separable subexpressions that could be counted. Here it is useful to think of what it means to say that a term occurs twice in a sentence. The term 'Goethe' can be thought of as having only one occurrence in 'Goethe influenced himself', or as having two occurrences, even though the expression only appears once. What is the difference? The difference consists in what substitutions are considered as applicable. If this sentence can only be turned into such other sentence as 'Gibbon influenced himself' and 'Carlyle influenced himself', then the term occurs only once, and the simple predicate involved is '... influenced himself'. If it can be turned into 'Goethe influenced Carlyle' and into 'Carlyle influenced Goethe', as well as 'Carlyle influenced himself', then the term occurs twice, and the simple predicate involved is '... influenced \_\_\_\_'. This consideration motivates defining the number of occurrences of a specified term in a sentence as the maximum number of sequential, non-interfering single substitutions substituting for that term that can be non-vacuously applied to the sentence. They are to be sequential, in that the second transform is to be applied to a result of applying the first, and so on in a chain of substitutions. By 'noninterfering' is meant that the substituting term of each transform does not contain the substituted term they all share, (see Appendix, K for how to recognize containing terms), a condition necessary to ensure that the substitutions used to count do not reintroduce the term being counted. Since each transform in such a chain must be non-vacuous, each must eliminate at least one occurrence of the term each transform substitutes for. So the maximum length of such a

chain will be the number of occurrences of that term in the sentence. Only sentences in which this number is finite for every occurring term are to be considered. [see Appendix, G].

The first stage in the discrimination of singular term occurrences was the identification of de jure symmetric inferences that could be thought of as substitutions of one term for another according to an identity commitment. At this stage singular terms could be picked out by saying which sentences they occurred in, and substitution transformations could be identified according to which terms they substituted for and which terms were substituted for them. At the next stage the requirement was added that each sentence have only a finite number of terms occurring in it, and that each of those terms occur only a finite number of times in any sentence. This brings the subsentential structure discerned by substitutional analysis closer to that of standard term-predicate prequantificational syntax, but it still falls far short. For notice that although the resources necessary to say what terms occur in a sentence and how many times they do have been considered, nothing so far has been said about *where* they occur. 'Goethe influenced Carlyle' and 'Carlyle influenced Goethe' both exhibit the same predicates, and the same terms, and those terms occur the same number of times in each. Yet they must be distinguished by semantically significant syntax, since they are distinguished semantically. One is true and the other is not, they are involved in different good inferences, and they have different incompatibilities. The ability to talk about what *position* a term occupies in a predicate is clearly essential to the specification of the *complex* predicates Frege showed the necessity of discerning in order to codify predicate inferences with quantifiers. For complex predicates add to the machinery of simple predicates precisely the capacity to cross-reference predicate positions, specifying which of them must be filled by occurrences of the *same* term.

In standard compositionally expressed syntax, this difficulty is handled by incorporating in each predicate an arbitrary linear order of term occurrences. Positions can then be specified numerically, and cross-referencing established by specifying that, for instance, the  $i^{\text{th}}$  and  $j^{\text{th}}$  positions must exhibit two occurrences of the same term. Such a linear order according to which term positions are identified and individuated is a syntactic artifact, however. It does not reflect any sort of linear ordering that must exist in the phenomena to which predicates are semantically linked. Nothing about the relation of influencing dictates a linear order of precedence. Or rather, the temporal ordering that might spring to mind here will not

\* Thus "Tacitus influenced Gibbon" is distinguished from "Gibbon influenced Tacitus" in that the former has 'Tacitus' in the 'Goethe' position and 'Gibbon' in the 'Carlyle' position, while the latter has 'Tacitus' in the 'Carlyle' position and 'Gibbon' in the 'Goethe' position.

do for admiring, and the order of activity one might invoke there will not do for married, and so on. Clearly one does not want to have to maintain that there is something about the relation being expressed by a predicate that dictates a linear order of term positions within that predicate. The numbering or ordering of predicate positions is a dispensable trick, but distinguishing positions which are reidentifiable across congruent sentences, and which can indicate different roles objects play in the relation they share, is not. How else might it be accomplished?

One solution is to index those predicate positions using an index set that is not linearly ordered. Pick an arbitrary representative of a simple predicate congruence class, say "Goethe influenced Carlyle," and use the terms occurring in it as the indices. Instead of talking about the first and second argument places, talk about the 'Goethe' and the 'Carlyle' positions of the *canonical congruence representative* "Goethe influenced Carlyle." Thus "Tacitus influenced Gibbon" is distinguished from "Gibbon influenced Tacitus" in that the former has 'Tacitus' in the 'Goethe' position and 'Gibbon' in the 'Goethe' position. The choice of which sentence in the congruence class to take as canonical representative is arbitrary, but no extraneous structure (such as a linear ordering) need be imported from outside. What is needed to make this work is a *position function*, by which is meant a function that takes as arguments any sentence, say 'Tacitus influenced Gibbon', and any diverse canonical representative of the congruence class of that sentence, say 'Goethe influenced Carlyle', and any term occurring in that canonical representative, say 'Goethe' (recall that terms are officially represented by sets of sentences in which they occur), and yields as result a term that occurs in the initial sentence at the *same* position that the specified term occupies in the canonical representative, in this case 'Tacitus'. Defining position functions in this way corresponding to each simple predicate then suffices for the definition of all of the complex predicates corresponding to each simple predicate, by cross-referencing on the positions they pick out. Thus the complex predicate '... influences himself' can be conceived as the class of congruent sentences in which the term occupying the 'Goethe' position is the same as the term occupying the 'Carlyle' position.

To do its indexing job properly, the canonical representative invoked by a position function should meet three conditions. First, it must present distinct indices, that is, the terms occurring in it must be *maximally diverse*: no term must appear more than once in it. Second, it ought to have no terms in common with the target sentence it is to index, i.e., the canonical

representative and the target sentence should be *term-diverse*. Finally, it simplifies things to require that all the terms occurring in canonical representatives should be *atomic*, that is, not exhibit terms in which other terms occur. The Appendix (K) shows how these can be recognized. (Without this stipulation a position function might be asked to say what term in 'Tacitus influenced Gibbon' occupies the position that 'Goethe' does in 'Goethe's wife influenced Carlyle'.) The notion of canonical sentences, as purely atomic and maximally diverse, permits a move one stage further towards standard subsentential syntactic analysis by showing how to eliminate *multigrade* predicates, i.e., those that do not have a definite number of terms to which they apply. The number of terms occurring in a canonical representative of a congruence class can be used as a measure of the *adicity* of the simple predicate defined by that class. Maximal diversity ensures at least one distinct term per position, and atomicity ensures at most one. For standard syntax, then, it must be the case that the number of terms occurring in each canonical representative of a congruence class is the same [see Appendix, H].

It remains to define the position functions. This requires being able to hook up occurrences of terms in arbitrary sentences with occurrences of terms in canonical representatives of the congruence classes of those sentences. The substitution transformations permit such cross-identification. "Tacitus influenced Gibbon" has 'Tacitus' in the 'Goethe' position of "Goethe influenced Carlyle," and 'Gibbon' in its 'Carlyle' place, because the former can be transformed into the latter by substituting 'Goethe' for 'Tacitus' and 'Carlyle' for 'Gibbon'. The information needed is provided by indexing of simultaneous substitutions by sets that match up each substituting term drawn from the terms the substitution applies to with the term substituted for it, drawn from the terms the substitution results in. To index the positions of a target sentence by the terms occurring in a congruent canonical sentence, pick a straightforward simultaneous substitution that applies to the canonical sentence and results in the target sentence. The output of the position function for any term appearing in the canonical sentence can then be taken to be the term that is matched with it by the index of that transform [see Appendix, I].

To provide proper positions by this recipe, the substitutions must be sufficiently well behaved that the construction is not sensitive to choice of canonical representative, or to which substitutions are employed to connect the target sentence to a canonical representative. If they are well behaved, then each n-adic predicate congruence class of sentences will be

associated with  $n$  functions assigning each sentence, whether or not canonical, a term occurring in it at the position defined by that function. It is satisfaction of this condition (stated in detail in the Appendix) that constitutes the possession by predicates of argument places that are fully definite. Not only is it settled which terms the predicate is applied to, and how many times each appears, but also what reidentifiable role each appears in is defined if the substitution transforms permit the cross referencing achieved when position functions are definable.

The capacity to define a position function corresponding to a congruence class makes it possible to define all of the complex predicates associated with that simple predicate. For instance, one can define the property of having the same term appear in both the 'Goethe' and the 'Carlyle' positions of a sentence congruent to 'Goethe influences Carlyle'. A different complex predicate is indicated as that shared by sentences congruent to the canonical "The influence of Tacitus on Gibbon is greater than that of Goethe on Carlyle," in which the 'Tacitus', the 'Gibbon', and the 'Carlyle' places are all occupied by occurrences of the same term, for instance "The influence of Goethe on Goethe is greater than that of Pindar on Goethe." All the cross-referencing of terms needed to generate the complex predicates can be specified in this way. More generally, from position functions it is straightforward to define operations of *variation* and *instantiation* corresponding to the Fregean operations of *omitting* and *replacing* a term. What corresponds to the metaphorical omission of a term from a sentence is a class of congruent sentences that agree in all but one term position, and include all the possible variations on the remaining term position. 'Omission' of more than one term is just variation on more than one term position. Replacing an omitted term with another is just choosing out of this set of variants the one that has the substituting term in the position of variation. All of this the position function makes possible. Van Fraassen has shown that once variation and instantiation can be defined, the full quantificational apparatus of complex predicates (what he calls 'selectors') is available [see Appendix, J]. So at this point the full subsentential syntax presupposed by the first order quantificational languages beloved of classical model theory has been made available.

In terms of the categorial grammars, the behavior of items of the basic category  $S$  of sentences has been presupposed. The position functions defined in terms of that behavior suffice to discern the occurrence and determine the syntactic behavior of items of category  $T$ , and those of the derived categories  $(T>S)$ ,  $((T,T)>S)$ , and so on, that is, predicates of

different adicities. With an eye to categorial generalization, room has been left at each stage for treatment of the categories  $(T>T)$ ,  $((T,T)>T)$ , and so on of functors that form terms from terms. All that is required is an extension of the notions of congruence, canonical representative, and position function from sentences containing terms to terms containing terms. It ought to be clear at this point how such an extension would go [it is elaborated in Appendix, K].

A similar example is provided by the categories  $(S>S)$ ,  $((S,S)>S)$  etc. of operators that form compound sentences out of sentential components. Here as before one would need to start with recognizable inferential patterns from which substitutional relations could be extracted. Since the sentential sign designs to be treated as expressing sentential compounds are not understood as literally containing or pretheoretically exhibiting occurrences of other sentential sign designs, embedded sentences will have to be extracted substitutionally, as terms were, and so represented by sets of (compound) sentences in which they will be said to figure as components. This is the way to treat the component or argument categories in the substitutional reconstruction of derived categories generally. An embedded sentence occurrence can be identified with what is intuitively the very same since the free-standing sentence will appear in the set of compounds containing the sentence, as a special degenerate compound. Congruence classes corresponding to simple sentential operators are then definable by substitutional accessibility, as before. Canonical representatives of the congruence classes are sentences containing only diverse sentences that do not themselves contain sentences, and position functions indexed by those canonicals are then definable subject to the same sort of conditions as were invoked for the case of terms and predicates. By these means the inferentially significant substitutional relations among sentences make available in principle the whole subsentential structure codified in categorial grammars, quite independently of any features of the sentential sign designs save their discriminability.

So neither need syntax recapitulate orthography, nor vice versa. Frege showed that syntactic structure must be distinguished from the orthographic structure by which a sentence or sentential sign design is built up out of its parts. The discovery of complex predicates showed that the categorial demands of inferentially significant syntax outrun what is provided by consideration only of the composition of sentential expressions out of subsentential expressions. By tying his notion of a function to the substitutional relations among sentences, and explaining these latter in terms of

the construction of sentential expressions, he showed how the richer syntactic categories required by inferential practice, paradigmatically complex predicates, could nonetheless be based on features of expressions that derive from their construction out of subexpressions. But Frege's way of conceiving those functions depends essentially on the possession by sentential sign designs of a compositional structure supporting 'omission and replacement' of their physical parts. The construction presented here shows how the variation and replacement that provide the structural core of this metaphor can be conceived in abstract substitutional terms, and how those substitutional relations can be extracted from the inferential practices governing the use of the expressions so related.<sup>15</sup> Syntactic structure can be imputed to sentential expressions completely apart from any facts about their constitution. So sign designs need not bear the physical stigmata of the syntactic structures they exhibit. Being used in a certain way is sufficient to count as displaying a subsentential structure that includes the occurrence of singular terms and the application to them of complex predicates. The composition of a sentential sign design, whether conceived of in terms of component pieces or physically specifiable features, is entirely inessential to its possession of complex syntactic form. To say that a particular singular term or predicate occurs in a sentence is to talk not about the construction of the sign designs that express that sentence, but about the use of the sentence in relation to other sentences. That the term occurs at a certain position in each of the complex predicates the sentence exhibits is the fact that the inferential relations that sentence stands in exhibit a substitutional structure of a certain kind.<sup>16</sup>

This having been said, it should be admitted immediately that it is not simply a bizarre coincidence that the languages we are familiar with maintain basically compositional relations between physical orthography and syntax. Reflecting syntactic structures in the physical composition of sign designs, while dispensable in principle, is extremely convenient in practice. Indeed, talk of 'convenience' may seem a substantial understatement of its importance. The inferential practices of a community, perhaps as internalized by an individual, have simply been taken for granted by the process recounted above whereby syntactic structure is extracted from or imputed to the sign designs by means of which individuals undertake and attribute inferentially articulated commitments. This essay has not addressed the question of how an individual might manage to catch on to these practices. To do so, he or she clearly must be able to distinguish, in a given context of attributed commitments, the claims that follow from

a given claim from those that do not. If this capacity is not to be attributed to sympathetic magic or simple good luck, it is hard to see how to avoid explaining it in terms of physical features of the sentential sign designs expressing those claims, which interlocutors can be trained to respond to differentially, and from which the syntactic structure implicit in those sign designs can be inferred. Suppose the sentential sign designs were numerals. Is it possible that the set of sentences substitutionally congruent to a given sentence should form a set that is not recursively enumerable? Provided the substitution transforms are otherwise well-behaved, such a possibility poses no problems for the construction of canonical representatives and position functions. But it would remain mysterious how the speakers of such a language were able to do so, were able to behave in such a way as to confer these syntactic structures and semantic significances on sign designs so related. It would be as mystifying as supposing that interlocutors reliably respond differentially to sign designs that are as far as we can tell indistinguishable by their perceptual apparatus.

No doubt it is appreciation of this point that has led to the widespread confusion between the syntactic structure and the orthographic composition of sign designs. But it is a cause rather than a reason for failing to distinguish how the trick is done from what doing it consists in, the mechanisms which organisms with definite abilities and disabilities can employ to achieve a certain end or master a certain practice on the one hand, and what counts as achieving the end or mastering the practice, on the other. The concern of this essay has been with the structures the use of sentential expressions must exhibit for it to be correct to say that singular terms and complex predicates occur in them. Consideration of how organisms with particular sorts of capacities might contrive to use expressions in this manner is another inquiry entirely. Language users do treat a term as occurring in a sentential sign design *because* of the presence of a physical component or feature. But what it is for that orthographic feature to have the syntactic significance of an occurrence of a singular term is for that sign design to be related to other sign designs according to inferential practices that can be understood as instituting substitutional relations on those sign designs, regardless of what mechanisms make those practices masterable by beings of a certain kind.

## APPENDIX

### A. *Eliminating substitution of simple predicates*

Synonymous predicates will be eliminated by the requirement that all sets of pairs of terms be represented as indices of some transformations, further along (the Index Plenum conditions). For 'Tacitus' can't appear substituted for '\_\_\_ influenced by ...', even if the latter had something that is everywhere replaceable for it, preserving the attribution of assertional commitment. Stepping back a little farther, although predicates are sometimes involved as the primary subjects of variation and instantiation in symmetric inferences, they are always also involved as substituends in asymmetric weakening inferences, in a way that singular terms never are. What about substitutions for or 'omissions' of other parts of speech, such as articles, subordinating conjunctions, transformation markers, etc.? Given the coarseness of the inferential net through which the substitutions are here filtered, these will all count as altering the predicate applied to the singular terms, and they are discussed under that heading just below. Notice that the vast majority of such substitutions would not be associated with symmetrical inferential significances.

### B. *Form of the Mappings*

Given a domain  $S$  of sentences, consider a set  $F$  of one-many mappings relating sentences to sentences. That  $f(p,q)$ , for some  $f$  in  $F$  and  $p,q$  in  $S$ , is to mean that there is an attributable set of identity commitments in the context of which anyone committed to  $p$  is committed to  $q$ . By the way these have been picked out it has been guaranteed that each relation will be transitively closed, (if  $f(p,q)$  and  $f(q,r)$  then  $f(p,r)$ ) and have a symmetrical inverse (so that  $f^{-1}(q,p)$  if  $f(p,q)$ ). That one sentence may be associated by a mapping with a number of distinct sentences is a consequence of their origins in extending attributed commitments. The transformation that corresponds to substituting 'the father of German drama' for 'Goethe' will apply to 'Goethe influenced Goethe', to yield not only 'the father of German drama influenced the father of German drama,' but also 'the father of German drama influenced Goethe', and 'Goethe influenced the father of German drama', because anyone committed to the first of these and to 'Goethe is the father of German drama' is committed thereby to all the rest. This can be put by saying that the substitutional relations are not conceived of as saturated, according to a requirement that every occurrence of the substituted for expression be replaced by an occurrence of the substituting expression. Rather, all of the incremental substitutional

results are considered, in which every combination of substituted for and substituting terms yields a sentence related to all the others by the substitution transformation or mapping. For all of the incremental substitutions correspond to inferential consequences of the original sentence, in the context of the identify commitments used to extract the substitutional relations.

### C. *Congruence*

The sentence  $s$  is *predicate congruent* to  $s_n$  in case there is a finite sequence of transforms  $\langle f_1, \dots, f_n \rangle$  and a sequence of sentences  $\langle s_1, \dots, s_{n-1} \rangle$  such that  $f_1(s, s_1)$ , and for each  $i$  through  $n$ ,  $f_i(s_{i-1}, s_i)$ . The sequence  $\langle f_1(s, s_1), \dots, f_n(s_{n-1}, s_n) \rangle$  is a chain *connecting* or *validating* the connection between  $s$  and  $s_n$ . Predicate congruence is an equivalence relation: it is reflexive since the validating chain might contain only vacuous connections, it is symmetric because of the existence of inverses, and it is transitive because of the transitivity of formation of finite validating chains. The role played by simple predicates in compositional instantiations of standard syntax is reconstructed here by equivalence classes of sentences under the predicate congruence relation. The success of such a reconstruction depends on there being 'enough' mappings. Conditions ensuring this are formulated below.

### D. *Picking out Single Substitution Transforms, Application*

In the compositional paradigm, if  $f$  substitutes 'Gibbon' for 'Tacitus' and 'Carlyle' for 'Goethe', it will apply non-vacuously (henceforth just 'apply') to any sentences to which any transform that substitutes only for 'Tacitus' applies, *and* to any sentences to which any transform that substitutes only for 'Goethe' applies. The prime complication involves terms that contain other terms, such as 'Aristotle's teacher'. The set of sentences to which transforms substituting for this term apply will be a proper subset of the set of sentences to which transforms substituting for 'Aristotle' apply. So minimality will not do in the general case to pick out all and only the single substitutions. Properties of substitution in the familiar compositional case point to the further conditions required to discriminate transformations that substitute for a single term that is contained in another term from transforms that make multiple simultaneous substitutions. This in turn shows how to formulate a condition on the well-behavedness of the initial set of transforms sufficient to ensure that discrimination by this means will be possible.

The *application sets*  $\{A(f) \mid f \text{ in } F\}$  induced by a set of transforms  $F$  are the sets of sentences such that some transform applies to exactly that set of sentences (i.e.,  $A(f) = \{s \text{ in } S \mid f(s, s'), \text{ for some } s' \text{ distinct from } s\}$ ). In the compositional case, if  $A(f)$  contains  $A(f')$  as a proper subset, then one of the terms  $f'$  substitutes for is contained in one of the terms  $f$  substitutes for, or  $f$  substitutes for more terms than  $f'$  substitutes for. This latter condition holds if there is another transform  $f''$  such that  $A(f'')$  does not contain  $A(f')$ , but such that  $A(f') \cup A(f'') = A(f)$ . If so, say that  $A(f)$  is a *proper sum* of  $A(f')$  and  $A(f'')$ . The transforms to be picked out as substituting for only one term are those whose application sets  $A(f)$  are minimal in the sense that none can be exhibited as the proper sum of an application set with any application set that is a proper subset of  $A(f)$ . If so, say that  $\text{SING}_F(f)$ , and that  $f$  is a 'single substitution'. In virtue of this definition, the application sets of the single substitutions form a basis for the whole set of application sets, in that each application set can be exhibited as the union of the application sets of some single substitutions. The first condition on  $F$  ensuring enough substitutions (COND1: *Basic Applications and Results*) is that the set of application sets be closed under formation of unions of the application sets of the single substitutions (which entails closure under all unions). The second condition ensuring that there are enough simultaneous substitutions (COND2: *Direct Connection*) is that if  $p$  and  $q$  are predicate congruent, then there is an  $f$  in  $F$  such that  $f(p, q)$ . It is stipulated further that there is always such an  $f$  that is *straightforward*. The concepts necessary to define straightforwardness will be developed in section H below. Here it is enough to register that the direct connection that is assured has special properties. Henceforth, attention is confined to sets of transforms that meet these conditions.

#### E. Results, Enough Transforms, Indexing Transforms, Defining Terms, and Occurrences of Terms in Sentences

Each *result set*  $R(f) = \{s' \text{ in } S \mid F(s, s') \text{ for some } s' \text{ distinct from } s\}$ . If the substitutions are to be indexible according to the substituting terms and the terms substituted for, then the set of result sets must be just the same as the set of application sets. To ensure that there are enough mappings, it is helpful to impose a stronger condition (COND3: *Index Plentitude for Single Substitutions*) on the set  $F'$  of transforms extracted from inferential practice: for any  $f, f'$  in  $F$ , there is an  $f''$  in  $F$  such that  $A(f'') = A(f)$  and  $R(f'') = A(f')$ . This will entail that for any  $f$  in  $F$  there is an  $f'$  in  $F$  such that  $A(f) = R(f')$ , and for any  $f$  there is an  $f'$  such that  $R(f) = A(f')$ .

The set  $T$  of terms induced on the sentences  $S$  by the transforms  $F$  can be taken to be  $\{A(f) \mid \text{SING}_F(f)\}$ , or equivalently  $\{R(f) \mid \text{SING}_F(f)\}$ . Each sentence is then associated with a set, possibly empty, of terms that occur in it, that is, sets of sentences  $T(s) = \{A(f) \mid \text{SING}_F(f), \text{ and } s \text{ is in } A(f)\}$ . It follows, given COND1, that if any  $f$  applies to  $p$  and not to  $q$ , there must be some term in  $T(p)$  that is not in  $T(q)$ .

Each single substitution  $f$  in  $F$  can be indexed by a pair of terms  $I(f) = \langle A(f), R(f) \rangle$ , the term substituted for and the term substituting for it. It follows from these definitions that the substituted term occurs in exactly the sentences to which the single substitution applies, and the substituting term occurs in exactly the sentences in which that substitution results. So one could equivalently identify terms with pairs of sets of mappings: those that substitute something for it and those that substitute it for something. Such a term 'occurs in' a sentence if one of its first set of mappings applies to it and one of its second set of mappings results in it. Distinct mappings may share an index antecedent or substituted for term, provided different index consequents or substituting terms are involved, and dually for substituting the same term for diverse antecedents. But the task in hand has no use for distinct single substitutions that share an index. This would presumably arise if one of the coindexical mappings did not recognize all of the incremental substitutions (those in which both substituting and substituted for term occur) So require (COND4: *Maximal Incrementality*) that each set of coindexical mappings contain one that is maximal, in that it connects two sentences if any mapping with the same index does so. Henceforth, only the maximal representative of each coindexical class of single substitutions is considered.

One term contains another term just in case every sentence in which the containing term occurs is one in which the contained term occurs. 'Goethe' occurs in every sentence in which 'Goethe's mother' occurs, though not vice versa (if they are to count as distinct terms). Two terms are containment related to one another if one contains the other (including each term's trivial containment of itself). The next condition on the behavior of substitution transforms makes reference to term containments. It specifies that substitutions leave alone occurrences of terms that are not containment related to the substituted for and substituting terms of the substitution index, and that they do alter those that are related. Say that  $f$  *introduces*  $t$  into  $p$  iff  $t$  is not in  $T(p)$  and for some  $q$  such that  $f(p, q)$ ,  $t$  is in  $T(q)$ , and that  $f$  *eliminates*  $t$  from  $p$  iff  $t$  is in  $T(p)$  and for some  $f$ -accessible  $q$ ,  $t$  is not in  $T(q)$ . Then COND5, *Introduction and Elimination*, requires that for any

f in  $SING_F$  with index  $\langle a, b \rangle$  (a and b not containment related), and for any t in T, f introduces t iff t is containment related to b, and f eliminates t iff t is containment related to a. The parenthesis is needed because if a contains b, there is no sentence containing a and not b, so if t is contained in b, no sentence that contains a can have t introduced into it. But f only applies to such sentences. Dually, t cannot be eliminated if b contains a and a contains t.

Given the decomposition condition specified below, the stipulation of COND5 for single substitutions has as a consequence its validity for simultaneous substitutions as well. Next, COND6 *Conservation of Embedding* says that if the index of f is  $\langle a, b \rangle$  (a and b not containment related), and  $f(p, q)$ , then b cannot occur in q more deeply embedded than it does in p. Depth of embedding or containment is defined recursively: if t is in  $T(p)$  and no  $t'$  in  $T(p)$  contains t, then the depth of embedding of t in p is at most 0. If in  $T(p)$  there is a term that contains t, but no term that contains a term that contains t, then the depth of embedding of it in p is at most 1, and so on. Again, stipulating this condition for single substitutions has the effect of enforcing it for all of them, in the context of the other conditions.

#### F. Indexing Simultaneous Substitution Transforms

The task is to assign each simultaneous substitution an index that will be a set of indices of the sort associated with single substitutions. The idea is to isolate the effects of each of these component indices by restricting attention to test sentences chosen to make the presence of the other component indices irrelevant. Such a procedure is legitimate only if the behavior of a simultaneous substitution mapping with respect to such test sentences is representative of its behavior with respect to the rest of the sentences. This is a matter of the way in which simultaneous substitutions relate to single substitutions. For the simultaneous substitutions to be *indexible*, they must meet some special conditions.

First, define  $TA(f)$ , the set of *terms* to which f applies, as the largest set of terms, the occurrence of any one of which in a sentence is sufficient for f to apply to that sentence (i.e., such that t is in  $TA(f)$  iff for any p [t is in  $T(p)$  iff p is in  $A(f)$ ]), and dually for  $TR(f)$ , the set of terms in which f results. Then impose COND7 *Decomposition*: for every substitution f in F, there is a set of single substitutions  $D(f) = \{g_1, \dots, g_n\}$  with the following properties: i) for all  $g_i$  in  $D(f)$ , there are t in  $TA(f)$ ,  $t'$  in  $TR(f)$  such that  $I(g_i) = \{\langle t, t' \rangle\}$  and ii) for all p, q in S,  $f(p, q)$  iff there is a congruence

validating chain of the form  $\langle h_1(p, x_1), h_2(x_1, x_2), \dots, h_m(m_{m-1}, q) \rangle$  connecting p and q, where each  $h_i$  is some  $g_j$  in  $D(f)$ . So each simultaneous substitution is to be *connection equivalent* to a set of single substitutions, that is, is to establish just the same connections among congruent sentences as are established by arbitrary sequences (with repetitions) of single substitutions from the correlated set. Although in the compositional case it is true that there are such decompositions, they do not need to be unique. This is so because addition of some single substitutions to the decomposing set may be redundant due to the transitivity of finite sequence formation. Sequences of applications of substitutions with indices drawn from  $\{\langle a, b \rangle, \langle b, c \rangle, \langle a, c \rangle\}$  will establish just the same connections as sequences drawn from single substitutions with indices in  $\{\langle a, b \rangle, \langle b, c \rangle\}$ , and similarly for  $\{\langle a, b \rangle, \langle u(a), c \rangle, \langle u(b), c \rangle\}$  and  $\{\langle a, b \rangle, \langle u(a, c) \rangle\}$  (where  $u(a)$  is a term containing (a)). For technical reasons it is best to deal only with sets of single substitutions that are closed under the operation of adding connection redundant single substitutions, so it is required that there always is such a one, and it is identified with  $D(f)$ . Since all the decompositions must be connection equivalent and restriction (see below) is to hold, they must in any case have many of their single substitutions in common. COND8 *Index Closure*: Within the set of decompositions of any f, there must be one that is maximal, in that for any index of a single substitution occurring in any decomposition, there is a single substitution with that index in the maximal decomposition. The *index*  $I(f)$  of the simultaneous substitution f is then defined as the union of the indices of the single substitutions that appear in its maximal decomposition  $D(f)$ .

Since the concatenation of two sequences all of whose elements are drawn from a given set is itself a sequence all of whose elements are drawn from that set, it follows from decomposability that simultaneous substitutions are transitive, in that if  $f(p, q)$  and  $f(q, r)$ , then  $f(p, r)$ , as noted above. More important, it follows from the decomposability of simultaneous substitutions that the Principle of Index Restriction holds: if  $I(f')$  is a subset of  $I(f)$  and  $f'(p, q)$ , then  $f(p, q)$ . It is now possible to state the stronger condition ensuring the existence of enough substitution mappings. COND9, *Index Plenitude for Simultaneous Substitutions* (compare COND3), says that for any set of single substitutions there is a simultaneous substitution that is connection equivalent with that set. Given this condition, Index Restriction can be strengthened so that  $f(p, q)$  iff  $f'(p, q)$ , whenever the index of  $f'$  is just the index of f restricted to index components whose antecedent or first element is in  $T(p)$ . These relations show that the behavior

of simultaneous substitutions meeting the conditions laid down is sufficiently regular that indices may be determined component by component, by considering their effect on test sentences in which only a single element of  $TA(f)$  occurs, as the text suggests. More specifically, start with the terms in  $TA(f)$  that contains no other terms in  $TA(f)$ . For each such  $t$ , pick a sentence  $p$ : i) in which  $t$  occurs, ii) in which no term containing  $t$  occurs. It is shown below that the existence of such test sentences in every case is guaranteed by COND12 (Enough Canonicals), in the context of the other conditions. Then by Conservation of Embedding and the Introduction and Elimination conditions, any term that is introduced into  $p$  by  $f$  (i.e., that it occurs in  $q$  but not  $p$ , for  $q$  such that  $f(p,q)$ ) and is not contained in any other term introduced by it, must correspond to a term that is paired with  $t$  as the consequent of an index component of  $f$ . After assigning sets of index components to each such uncontained  $t$  in  $TA(f)$ , proceed recursively to terms in  $TA(f)$  that contain only terms that do not contain other terms in  $TA(f)$ . Stop this procedure when the set of index components assembled generates all the congruence connections established by  $f$ , as they are guaranteed sooner or later to do. In this way the indices of all the simultaneous substitutions are determined.

#### G. Finite Term Occurrence

Call the dual requirement that there be only a finite number of terms in each sentence, and that each one occur only a finite number of times the '*Finite Term Occurrence Condition*' (COND10). It entails that in each sentence term embedding proceeds only to a finite depth.

#### H. Adicity

This is COND11: *Fixed Adicity*, which ensures that there is a fixed number of positions associated with each simple predicate. It is not entailed by the weaker Finite Term Occurrence Condition above. In the context of a language in which some terms contain other terms, a stronger version of COND11 is wanted, to the effect that each sentence in the congruence class must have the same number of *toplevel term occurrences*. It will be sufficient for a term  $t$  to have a toplevel occurrence in  $p$  if it is not contained in any other term in  $T(p)$ , so all terms have only toplevel occurrences in canonical congruence representatives, and this statement of COND11 entails the other. It remains to define toplevel occurrence where the term has multiple occurrences, only some of which are toplevel. It is necessary only to look at terms that occur more than once. Start with the contained-most terms in  $T(p)$ , i.e., those that are contained but do not contain others. By

CONDs 5 and 6, substitution for terms that occur only embedded cannot introduce an unembedded occurrence of the substituting term. So use test transforms in which  $t$  is substituted for and the substituting term is atomic and does not occur in  $p$ . If the application of any such transform results in a sentence in which the atomic index consequent is introduced uncontained, then  $t$  has a toplevel occurrence in  $p$ . If so, then apply a distinct test transform to the results of the previous application, to test for further toplevel occurrences of  $t$  in  $p$ . In this way the number of toplevel occurrences, if any, of each  $t$  in  $T(p)$  can be determined. COND11 requires that the sum of all of these over every term in  $T(p)$  must be the same for any congruent sentences. This adicity must be finite, by COND10.

Now it is possible to specify the definition of the term *straightforward*, which was left dangling in the statement of COND2 (that part of the condition has not been appealed to in the interim, so the hiatus is purely expository). A mapping  $f$  is a straightforward connection of  $p$  and  $q$  if i)  $f(p,q)$ , and ii) if  $\langle a,b \rangle$  is in  $I(f)$ , then  $a$  is in  $T(p)$  and  $b$  is in  $T(q)$ , and iii) the shortest chain validating the indexing decomposition by establishing the connection of  $p$  to  $q$  by sequential application of single substitutions with indices appearing as components in the index of  $f$  is of length equal to or less than the adicity of the congruence class of  $p$  and  $q$ . The first condition says that  $f$  connects  $p$  and  $q$ , the second that the antecedents of index components all appear in the sentence that  $f$  is applied to, and their consequents all appear in the sentence it then results in. The third condition is satisfied in the compositional case by a chain of single substitutions that move one by one through the positions of the predicate, substituting in each the term that occupies that position in  $q$  for the term that occupies it in  $p$ , with at most one substitution required per position.

It is important for the constructions involving them that there be enough canonical sentences. This requirement can be made more definite as COND12: *Enough Canonicals*: for any sentence  $p$ , there is a canonical sentence  $q$  that is congruent to  $p$ , and is term-diverse from it (that is,  $T(p)$  and  $T(q)$  are disjoint), and for any atomic term not occurring in  $p$ , there is a congruent, canonical, term-diverse  $q$  that contains it. This entails that there are enough atomic terms for the various constructions. Together with Index Plenum and Introduction and Elimination it also entails that there are enough test sentences to permit the discovery of the indices of the simultaneous substitutions, as in (F) above. By I&E those test sentences result from canonicals by substituting the test term for an arbitrary term, and by the plenum condition the single substitution required to discover

such a term exists and applies non-vacuously to the canonical.

### I. Existence of Position-Indexing Simultaneous Substitution

By Direct Connection, for any target sentence whose predicate positions are to be indexed and for any term-diverse canonical sentence congruent with it, there is a substitution  $f$  that straightforwardly connects them. So there is a shortest chain of applications of single substitutions with indices drawn from the index of  $f$ , of length no greater than the number of atomic terms in the canonical sentence, that connects the canonical and the target sentence. By the Introduction and Elimination condition, that chain must have at least as many steps as there are distinct atomic terms in the canonical. For each term in the canonical must be eliminated by some substitution, since the canonical representative and the target sentence are stipulated to be term-diverse. Where  $n$  is this adicity, the  $n$  single substitutions in that chain must also, by I&E and straightforwardness, have indices each of which consists of a different term appearing in the canonical, and whose consequents jointly include all and only the terms occurring in the target sentence. This set of ordered pairs is a *position matching* of the toplevel (by Conservation of Embeddedness) target sentence terms with those of the canonical. Accordingly, the output of a position function that takes the target sentence, the canonical, and a term in the canonical as arguments, could be taken to be the toplevel term occurring in the target sentence that is matched with the test term chosen from the canonical.

The remaining requirement is that all the different position matchings of target sentences and diverse canonicals be in agreement. This means that the positions of a predicate congruence class of adicity  $n$  should be indexible by  $n$  sets of pairs of a canonical sentence and a term occurring in it, which will count as the term occupying that position in it. The position function should associate each target sentence and each such position with a single term occurring toplevel in the target sentence. This requires, COND13: *Position Equivalence*, that i) position matching be transitive within the class of canonical sentences and ii) if any two position matchings assign the same position (equivalence class of pairs of canonicals and terms occurring in them) to terms  $t$  occurring in target sentence  $p$  and  $t'$  occurring in target sentence  $q$ , then *all* position matchings assign the same position. Part (i) ensures that the canonical-term pairs are sorted into equivalence classes by position matching when target sentences are chosen exclusively from among canonical representatives. Reflexivity and symmetry follow from general considerations, so all that is required is that if  $t$  in canonical

$p$  is matched by some shortest path validating straightforwardness to  $t'$  in canonical  $q$ , and  $t'$  in  $q$  is matched to  $t''$  in  $r$ , then every matching of  $t$  in  $p$  to  $r$  must yield  $t''$ , which is (i). Part (ii) extends this equivalence to noncanonical target sentences.

### J. A Related Construction

“Quantification as an Act of Mind,” *Journal of Philosophical Logic*, 11, (1982) 343-369 (hereafter QAM). Van Fraassen’s project is very close in spirit to the present one, though the setting and constructions are quite different. Part of the justification for the hand-waving done at some stages in this construction lies in the mathematical sophistication with which QAM has worked out its related scheme. There are three reasons for thinking that this essay is not just doing badly what it does well. First, this story starts farther back in the philosophical story than QAM does. The latter treats variation and instantiation as primitives, saying only a little about how these procedures relate to the use of a language. For instance, that construction simply presupposes that the language has recognizable general propositions in it, where part of the concern here is to understand what this might mean. QAM’s primitives are constructed here only at the end of a long process of massaging the inferentially significant transforms that could be hooked up directly with linguistic practice. Second, the QAM account as it stands does not deal with term occurrences inside other term occurrences. The more realistic level of generality that permits the discerning of two occurrences of ‘Goethe’ in ‘Goethe’s father influenced Goethe’, is important for making plausible the claim that the substitutional approach can form a basis for a full categorial grammar, with derived categories of all sorts. Finally, QAM’s detailed and powerful metatheory of substitution algebras secures the claim *that* substitution transformations are sufficient to constitute subsentential syntactic structure at the cost of making it quite hard to see *how* different features of those transformations can be exploited to yield different sorts of syntactic structure. The present exposition has sacrificed some of the rigor that QAM makes available, for the sake of perspicuous presentation of how one recovers from substitutions such crucial features as repetitions, containments, and positions of terms. So the two approaches are complementary, not competitive.

### K. Congruence, Canonicals, and Position Functions for Compound Terms

To avoid distracting complication, in the text the possibility that one term contains another, as ‘Eric’s father’ contains an occurrence of ‘Eric’, has been deemphasized. In particular, positions have been defined only

for toplevel term occurrences, not for terms occurrences inside other terms, even though in the most general case complex predicates can be formed by 'omitting and replacing', that is varying and instantiating, such contained term occurrences. The extension to this case of the ideas already employed is straightforward, so only a broad sketch will be offered here. What is required is to extend the notions of congruence via substitutional relatedness, canonical representative, and position function, from sentences, in which substituting and substituted for terms occur, to terms, in which substituting and substituted for terms occur. Such an extension permits the definition of complex predicates formed by variation on term occurrences embedded to arbitrary depths in other terms occurring in the sentence, for it provides the resources to specify for each such occurrence what position it occupies in the term occurring toplevel that contains it, and what position that containing term occupies in the sentence. Thus sentences can be matched accordingly as they are alike everywhere except at that position, for instance. Instantiation of such variants by 'plugging in' a specified singular term is also definable, as before.

As was mentioned, containing terms can be recognized by inclusions of sets to which transforms nonvacuously apply:  $t_1$  is contained in  $t_2$  just in case  $t_2$  is a subset of  $t_1$ . According to the definitions offered above, this means that for any sentence  $s$ , and for any transforms  $f_1, f_2$ , such that  $\langle t_1, X \rangle$  is in  $I(f_1)$  and  $\langle t_2, Y \rangle$  is in  $I(f_2)$ ,  $f_1$  applies nonvacuously to  $s$  if  $f_2$  does, but perhaps not vice versa. What is it for two containing terms to be congruent? Intuitively, the terms 'Goethe's mother' and 'Carlyle's mother' are congruent, because one can be turned into the other by substituting a term one contains for a term the other contains, 'Carlyle' for 'Goethe' or vice versa. Substitutions are defined in the first instance on sentences, not terms, but since it has been settled what it is for terms to occur in sentences, and even to occur at particular toplevel positions, this can't be difficult. If  $f$  is a single sentential substitution mapping with index  $\langle a, b \rangle$ , then its extension to terms,  $f^*$ , with the same index, may be defined by  $f^*(t, t')$  iff  $t$  contains  $a$  and  $t'$  contains  $b$  and for  $g$  such that  $I(g) = \langle t, t' \rangle$  and for any  $p$  and  $q$ , if  $g(p, q)$ , then  $f(p, q)$  (and similarly for simultaneous substitutions). Intuitively,  $t = M(a)$  and  $t' = M(b)$ , for some functor  $M$  that forms terms from terms, so the effect of any substitution of  $M(b)$  for  $M(a)$  can always also be achieved by substituting  $b$  for  $a$ , as in the example above.

With the substitution mappings connecting terms in hand, it is possible to proceed with the definitions of congruence, canonical representative,

and position function essentially as before. Notice that it would be possible for the original sentential substitution mappings to be well-behaved with respect to the indexing of toplevel positions, and still fail to meet all the conditions required to discriminate distinct roles or positions within terms. In that case (some or all of) the term functors could only be understood as applying to *sets* of terms, not to terms playing distinct roles, that is like 'the author of X, Y, Z...', not as in 'the writer who first explained the influence of X on Y'. The one nontrivial complication that consideration of term functors does entail that does not have an analog in the treatment of predicates concerns differences in the depth of occurrence of terms. The toplevel positions considered previously all occurred at the same level, so the procedures employed there cannot simply be extended to define the positions of term occurrence in terms like 'the mother of the sculptor who most influenced Goethe's teacher'. Both congruence and canonical representatives will have to be defined recursively, for each level of containment, since that term, for instance is at one level congruent with 'the mother of Carlyle', and at others with 'the mother of the sculptor who most influenced Tacitus'. There is accordingly a congruence hierarchy, in which  $t$  is 1-congruent with  $t'$  iff for some  $f$  in  $F$  such that if  $\langle x, y \rangle$  is in  $I(f)$ , then  $x$  is properly contained in  $t$  and  $y$  is properly contained in  $t'$  and  $f^*(t, t')$ .  $t$  is 2-congruent with  $t'$  iff in addition  $t$  and  $t'$  are 1-congruent, and so on.

All terms have in effect been treated as at least level 0 terms, and their canonical representatives at that level as terms that contain no terms, that is, atomics. A term is, in addition, a level 1 term if it contains some other terms. Its canonical representative at that level contains diverse canonical representatives of level at most 0 (that is, atomic terms). Level 2 terms contain terms of level 1, and their canonical representatives contain diverse canonical representatives of at most level 1, and so on. It follows then that one can pick out any position at any depth in a compound containing term by picking a canonical representative of the containing term of the same level as the depth of the term occurrence in question, and indexing that occurrence by an atomic term occurring in that canonical. A fully general position function then applies to any term  $t$  and to any target term  $t'$  that it contains. For any canonical representative of it at the level to which  $t'$  is embedded in  $t$ , it returns an atomic term contained in the canonical that occupies just the same position in the canonical that the target term does in  $t$ . The matching of terms to construct the position function occurs just as before, subject to analogous conditions on the well-

behavedness of the terms and transforms. This method for recursively defining congruence and canonical representatives for arbitrarily containing and contained terms applies equally well to the case of sentences that contain sentences, or terms that contain sentences. It provides a recipe for generating, from inferentially identified substitution relations, position functions of the sort defined in this essay for syntactic structures of arbitrary derived categories in a full categorial grammar.

#### L. Summary Remarks on the Conditions

It will be helpful to say a few words about the thirteen conditions that have been laid down governing the substitution relations extracted from inferential practice and used to impute subsentential structure. The idea they subserve might be put as follows: suppose there were a compositionally instantiated syntax of the familiar sort governing the use of sentences, but that it is invisible to the theorist (the bricks have been painted over, obscuring the joints and so the construction). How could one work backwards to recover that subsentential structure? The answer is the construction that proceeds by defining predicate congruent sets of sentences, terms as sets of sentences, canonical representatives, and position functions. COND1-COND13 offer one set of sufficient conditions for the success of this construction of the syntactic structures taken for granted as raw materials by standard compositional orthographies, by means of position functions indexed by congruent canonicals. The same effect could be achieved in many other ways.

The conditions are of unequal status and function. COND1 is redundant (it follows from 9, 7, 3, and the definition of substitution indices). CONDS 4, 8, and 10 (Maximal Incrementality, Index Closure, and Finite Term Occurrence) are minor matters of detail and technical convenience. The functions of the remaining nine may be divided as follows. Conditions 2, 3, and 9 (Direct Connection and the two plenitude conditions) guarantee the existence of enough transforms. Condition 12 (Enough Canonicals), and 5 (Introduction and Elimination) guarantee the existence of enough sentences and terms, in the context of the previous conditions. Together these ensure that all the substitutions that can happen do happen.

It might be argued that one way of working backwards from inference to subsentential structure is better than another by showing that the more general notions of term and predicate occurrence that arise by omitting some subset of the conditions imposed have some independent interest. For this construction, the remaining conditions 5,6,7,11, and 13 are the

ones to look at, not having to do with the existence of enough transformations, sentences, and terms, but with the uniqueness or invariance of some property, or the well-behavedness of the substitutions. Introduction and Elimination makes it possible to associate single substitutions with terms substituted for and substituting for them. Without condition 6 (Conservation of Embedding) the term containment structure would be amorphous, with only toplevel occurrences of terms having positions reidentifiable across substitutions. Condition 7 (Decomposition), saying that all substitutions are connection equivalent to sets of single substitutions (Condition 9 presents the converse), specifies that the simultaneous substitutions behave appropriately with respect to single substitutions. Without it the restriction principle won't hold, and only single substitutions would be indexible. Condition 11 (Fixed Adicity) distinguishes proper predicates from multigrade predicates that apply to different numbers of arguments. Condition 13 (Position Equivalence) distinguishes proper predicates with positions indicating reidentifiable roles for terms from predicates that apply to equinumerous sets of terms.

#### NOTES

1. *Word and Object*, M.I.T. Press, 1960, pp. 90, 96. The conceptual role in question is an intralinguistic one, a matter of how an expression must be used so that the question of its relation to something represented is one that can be raised at all.
2. "General Semantics," in Harman and Davison (eds.) *Semantics of Natural Language* Reidel, Dordrecht, 1972.
3. Recall, however, that Chomsky showed that one should not expect to generate the well-formed sentences of natural languages by concatenation, combination, or tree-structuring of any set of categories of this sort. To any such "Phrase structure grammar" will have to be added transformations of such combinatory structures. Categorial classifications are just the raw materials for grammar in this sense, and don't have anything to say about how one might proceed to the rest of the task of syntax once one has the categories.
4. This exposition follows Dummett's setting-out of this point in chapter 2 of *Frege: Philosophy of Language* Harper and Row, New York, 1973.
5. See "On Concept and Object" and "Function and Concept," in Geach and Black's *Philosophical Writings of G. Frege* Blackwell, Oxford, 1970.
6. I develop an account along these lines in "Asserting" *Nous*, XVII 4, November 1983, pp. 637-650.
7. Discussed in a sequence of papers culminating in the definitive "Towards a Theory of Predication," in Bogen and McGuire (eds.) *How Things Are*, Reidel, Dordrecht, 1985, pp. 285-322.
8. This is not a goal that can be achieved by the simple expedient of defining the categorial dual of Jumblese, in which sentence inscriptions are inscriptions of simple predicate expressions, and features such as color and font encode the sequences of singular terms to which the predicates apply. Jumblese and its categorial dual each treat one sort of

subsentential category as corresponding to a kind of expression or sign design that could stand on its own, and occurrences of the other as relationally induced features. For this reason, each is only a half-way house on the road to exhibiting the dispensability of subsentential expressions of all sorts.

9. Jumblese exhibits an unusual relation between its syntax and its physical orthography, but it is not for that reason concerned with unrepeatable tokenings rather than repeatable types of expressions. The construction below likewise proceeds on the assumption that items of the syntactic category of sentence have already been associated with sign design repeatables. A full account would have to recognize that in any language containing indexical expressions some of these groupings would be generated not by orthographic cotypicality, but by anaphoric connections between unrepeatable tokenings. This 'complication' is of the first importance, but the point of this paper can be made without delving into it. For present purposes substitution relations can be thought of as involving sign designs instantiating repeatable sentences. So type-token niceties are suppressed in the text.
10. The function of logical vocabulary is explained as making explicit as the contents of assertional commitments what is implicit in inferential practice in the author's "Varieties of Understanding," in N. Rescher (ed.) *Reason and Rationality in Natural Science*, University Press of America, 1985.
11. These correspond roughly to showing that the substitution algebra being considered informally is, respectively, sound and complete, with respect to ordinary compositionally based substitution. It is usually easy enough to check the former as conditions are stated below, but real proof is exceptionally tedious, requiring as it does a full-blown apparatus to represent omission and replacement. It is accordingly omitted. See van Fraassen's careful treatment, cited in Appendix J. The latter is discussed more fully, for it is difficult to understand how it might work, even if one has become convinced, for instance by van Fraassen, that it is true.
12. One need only consider substitution for singular terms in order to be able to specify substitutions for simple predicates, so it is appropriate to focus on the former.
13. Elaborated in my "Asserting," op. cit. The construction presented below does not in any way depend on understanding inferential relations in this normative, social practical manner.
14. Building in this extensionality requirement for primary occurrence of terms is methodologically justified. For it amounts to using intersubstitutability *salva assertibilitate* as a criterion of primary term occurrence. If other sorts of term occurrences are to be discerned (e.g., secondary occurrences inside belief-attributing contexts), the warrant for doing so must be explicitly developed, along the same sorts of substitutional lines pursued here. But such a project will presuppose the success of the one involving primary or 'ground level' occurrences.
15. The Appendix offers a set of conditions on the substitution relations strong enough to ensure that the construction of position functions can proceed. Computer implementation of variants of the construction (in PROLOG on a personal computer) shows that in many cases the failure of the conditions to be true everywhere does not interfere with constructing position functions, so long as the generalizations they involve are 'true enough'.
16. That discerning occurrences of terms in sentential expressions does not require the existence of subexpressions that are terms was concluded from considerations of fundamental features of the inferential role played by the occurrence of singular terms in sentences. This conclusion does not automatically extend to their full conceptual role, which includes as well at least the possibility of deictic and anaphoric term uses, for instance. It was claimed above only that nothing can count as a (primary) term occurrence,

and so be available as such for deictic and anaphoric entanglements or significance, unless it plays the appropriate substitutional role in determining what follows from the claim in the sentential expression of which the term occurs, and what it follows from. Playing that role does not put any constraints on the construction of the sentential sign designs. But since playing the inferential role of a singular term is only a necessary, and not a sufficient condition of playing anaphoric or indexical roles, it does not follow that these roles put no constraints on the sentential sign designs in which terms occur. Extending the substitutional approach to the relation between sign designs and those further uses of singular terms lies outside the scope of this paper.